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Table of $(\lambda, 0) \times (4, 0)$ $SU_3 \supset R_3$ Wigner coefficients

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Abstract. In this paper, algebraic expressions for $(\lambda, 0) \times (4, 0)$ reduction Wigner coefficients in the $SU_3 \supset R_3$ physical basis are presented. They are obtained by a building-up process. These tables are useful in studies of nuclear algebraic models, such as the sdg interacting boson model.

The SU_3 group is one of the most extensively used groups in physics and chemistry. It is widely used in many branches of physics [1–9]. For instance, in the nuclear interacting boson model (IBM) [15], SU_3 has been widely used in the description of rotational motions in deformed nuclei. Many authors studied the SU_3 group in the Gelfand basis [10–14].

The Wigner coefficients in the $SU_3 \supset R_3$ physical basis are essential in many detailed calculations of the matrix elements in specific applications [16, 17]. Extensive tables of algebraic expressions have been given by Vergados [18]. These tables are sufficient for the sd shell model calculations and for the interacting boson model calculations involving only the s and d bosons. A computer program is also available to calculate these coefficients numerically [19, 20]. With the advent of vector coherent states (VCS) methods [21], the $SU_3 \supset R_3$ Wigner coefficients are constructed in VCS for $(\lambda, \mu) \times (2, 0)$, with arbitrary λ and μ [22].

In one of the extensions of the IBM, the g bosons are introduced to study the effects of hexadecapole degrees of freedom [23]. In order to gain an algebraic expression for the electric and magnetic transition rates for low-lying levels, one needs to know the algebraic expression for $(\lambda, \mu) \times (4, 0)$ Wigner coefficients. Although these tables of coefficients are very useful to physicists working in a specific field, the ones they need are usually not available. They have to calculate these needed tables themselves. These results are piecemeal and are scattered in various articles and are not easily available to perspective users. In many cases they are not complete and only for very specific cases. For instance, $(0, 4) \times (4, 0)$ tables can be found in [24]. $\langle (2N, 0)0, 0; (4, 0)0, L \| (\lambda, \mu), \kappa, L \rangle$ for $L = 0, 2, 4$ were given in [16]. $\langle (\lambda, 0)0, L_1; (4, 0)0, l \| (\lambda + 4, 0), 0, L \rangle$ was given in [18]. Since general algebraic expressions for the more general case of $(\lambda, 0) \times (\mu, 0)$ with arbitrary λ and μ are not easy at the moment [13], one has to obtain the tables with arbitrary λ but with specific μ. In this paper, we present algebraic expressions for $(\lambda, 0) \times (4, 0)$ Wigner coefficients. The direct motivation is to provide coefficients for interband transition studies in the SU_3 limit of the sdg IBM. We were able to study intraground state band transitions [25]. But our efforts were hindered by the lack of Wigner coefficients when studying interband transitions. Although for this interband transition study we need only

Table 1. The SU_3 Racah coefficients.

	(λ, μ)				
	$(\lambda_1+4, 0)$	$(\lambda_1+2, 1)$	$(\lambda_1, 0)$	$(\lambda_1-2, 3)$	$(\lambda_1-4, 4)$
	$U((\lambda_1, 0)(3, 0)(\lambda, \mu)(1, 0); (\lambda_{12}, \mu_{12})(4, 0))$				
$(\lambda_1+3, 0)$	1	$\sqrt{\frac{\lambda_1}{4(\lambda_1+3)}}$	—	—	—
$(\lambda_1+1, 1)$	—	$\sqrt{\frac{3(\lambda_1+4)}{4(\lambda_1+3)}}$	$\sqrt{\frac{\lambda_1-1}{2(\lambda_1+1)}}$	—	—
$(\lambda_1-1, 2)$	—	—	$\sqrt{\frac{\lambda_1+3}{2(\lambda_1+1)}}$	$\sqrt{\frac{3(\lambda_1-2)}{4(\lambda_1-1)}}$	—
$(\lambda_1-3, 3)$	—	—	—	$\sqrt{\frac{\lambda_1+2}{4(\lambda_1-1)}}$	1
	$U((\lambda_1, 0)(2, 0)(\lambda, \mu)(2, 0); (\lambda_{12}, \mu_{12})(4, 0))$				
$\lambda_1+2, 0)$	1	$\sqrt{\frac{\lambda_1}{2(\lambda_1+2)}}$	$\sqrt{\frac{\lambda_1(\lambda_1-1)}{6(\lambda_1+1)(\lambda_1+2)}}$	—	—
$(\lambda_1, 1)$	—	$\sqrt{\frac{\lambda_1+4}{2(\lambda_1+2)}}$	$\sqrt{\frac{2(\lambda_1-1)(\lambda_1+3)}{3\lambda_1(\lambda_1+2)}}$	$\sqrt{\frac{\lambda_1-2}{2\lambda_1}}$	
$(\lambda_1-2, 2)$	—	—	$\sqrt{\frac{(\lambda_1+2)(\lambda_1+3)}{6\lambda_1(\lambda_1+1)}}$	$\sqrt{\frac{\lambda_1+2}{2\lambda_1}}$	1

Table 2. $\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda+2, 1)L\rangle$.

	L_1	$\lambda - L =$ even
$l=0$	L	$\sqrt{\frac{4L(L+1)(\lambda-L+2)(\lambda+L+3)}{5\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
$l=2$	$L-2$	$-(\lambda-2L+4)\sqrt{\frac{6(L-1)(L+1)(\lambda+L+1)(\lambda+L+3)}{7(2L-1)(2L+1)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	L	$(12-4L-4L^2+3\lambda)\sqrt{\frac{(\lambda-L+2)(\lambda+L+3)}{7(2L-1)(2L+3)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	$L+2$	$(\lambda+2L+6)\sqrt{\frac{6L(L+2)(\lambda-L)(\lambda-L+2)}{7(2L+1)(2L+3)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
$l=4$	$L-4$	$-\sqrt{\frac{4(L-3)(L-2)(L-1)(L+1)(\lambda+L-1)(\lambda+L+1)(\lambda+L+3)(\lambda-L+4)}{(2L-5)(2L-3)(2L-1)(2L+1)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	$L-2$	$(36+4L-4L^2+9\lambda+2\lambda L)\sqrt{\frac{(L-2)(L-1)(\lambda+L+1)(\lambda+L+3)}{7(2L-5)(2L-1)(2L+1)(2L+3)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	L	$-(20-2L-2L^2+5\lambda)\sqrt{\frac{18(L-1)(L+2)(\lambda-L+2)(\lambda+L+3)}{35(2L-3)(2L-1)(2L+3)(2L+5)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	$L+2$	$(28-12L-4L^2+7\lambda-2\lambda L)\sqrt{\frac{(L+2)(L+3)(\lambda-L)(\lambda-L+2)}{7(2L-1)(2L+1)(2L+3)(2L+7)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	$L+4$	$\sqrt{\frac{4L(L+2)(L+3)(L+4)(\lambda-L-2)(\lambda-L)(\lambda-L+2)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)\lambda(\lambda+1)(\lambda+2)(\lambda+4)}}$
	L_1	$\lambda - L =$ odd
$l=2$	$L-1$	$\sqrt{\frac{3(L-1)(\lambda-L+3)(\lambda+L+2)(\lambda+L+4)}{7(2L+1)\lambda(\lambda+1)(\lambda+2)}}$
	$L+1$	$-\sqrt{\frac{3(L+2)(\lambda-L+1)(\lambda-L+3)(\lambda+L+4)}{7(2L+1)\lambda(\lambda+1)(\lambda+2)}}$
$l=4$	$L-3$	$\sqrt{\frac{(L-3)(L-2)(L-1)(\lambda+L)(\lambda+L+2)(\lambda+L+4)}{(2L-3)(2L-1)(2L+1)\lambda(\lambda+1)(\lambda+2)}}$
	$L-1$	$-\sqrt{\frac{9(L-2)(L-1)(L+2)(\lambda-L+3)(\lambda+L+2)(\lambda+L+4)}{7(2L-3)(2L+1)(2L+3)\lambda(\lambda+1)(\lambda+2)}}$
	$L+1$	$\sqrt{\frac{9(L-1)(L+2)(L+3)(\lambda-L+1)(\lambda-L+3)(\lambda+L+4)}{7(2L-1)(2L+1)(2L+5)\lambda(\lambda+1)(\lambda+2)}}$
	$L+3$	$-\sqrt{\frac{(L+2)(L+3)(L+4)(\lambda-L-1)(\lambda-L+1)(\lambda-L+3)}{(2L+1)(2L+3)(2L+5)\lambda(\lambda+1)(\lambda+2)}}$

the decomposition $(\lambda, 0) \times (4, 0) \supset (\lambda, 2)$ and $(\lambda, 0) \times (4, 0) \supset (\lambda+2, 1)$, we have also calculated the other two decompositions: $(\lambda-2, 3)$ and $(\lambda-4, 4)$ for completeness and potential users.

Table 3. $\langle(\lambda, 0)L_1; (4, 0)\|(\lambda, 2)\kappa, L\rangle$.

$\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda, 2)\kappa = 0L\rangle$

	L_1	$\lambda - L =$ even
$l = 0$	L	$(2\lambda^2 + 4\lambda - 3L - 3L^2)\sqrt{\frac{2(\lambda-L+2)(\lambda+L+3)}{15(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
$l = 2$	$L - 2$	$(12L - 6L^2 - \lambda + 6\lambda L + \lambda^2)\sqrt{\frac{(L-1)L(\lambda+L+1)(\lambda+L+3)}{7(2L-1)(2L+1)\lambda(\lambda-1)(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
	L	$-(\lambda^2 + 2\lambda + 9 - 6L - 6L^2)\sqrt{\frac{2L(L+1)(\lambda-L+2)(\lambda+L+3)}{21(2L-1)(2L+3)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L + 2$	$(-18 - 24L - 6L^2 - 7\lambda - 6L\lambda + \lambda^2)\sqrt{\frac{(L+1)(L+2)(\lambda-L)(\lambda-L+2)}{7(2L+1)(2L+3)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
$l = 4$	$L - 4$	$-(\lambda - L + 1)\sqrt{\frac{6(L-3)(L-2)(L-1)L(\lambda-L+4)(\lambda+L-1)(\lambda+L+1)(\lambda+L+3)}{(2L-5)(2L-3)(2L-1)(2L+1)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L - 2$	$(-7 - 4L + 2L^2 + 5\lambda - 2L\lambda + 2\lambda^2)$
		$\times\sqrt{\frac{6(L-2)(L-1)L(L+1)(\lambda+L+1)(\lambda+L+3)}{7(2L-5)(2L-1)(2L+1)(2L+3)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
	L	$-(-5 + L + L^2 + 2\lambda + \lambda^2)$
		$\times\sqrt{\frac{108(L-1)L(L+1)(L+2)(\lambda-L+2)(\lambda+L+3)}{35(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L + 2$	$(-1 + 8L + 2L^2 + 7\lambda + 2L\lambda + 2\lambda^2)$
		$\times\sqrt{\frac{6L(L+1)(L+2)(L+3)(\lambda-L)(\lambda-L+2)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L + 4$	$-(\lambda + L + 2)\sqrt{\frac{6(L+1)(L+2)(L+3)(L+4)(\lambda-L-2(\lambda-L)(\lambda-L+2)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-1)\lambda(\lambda+2)(\lambda+3)(2\lambda^2+8\lambda-L^2-L+8)}}$

$\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda, 2)\kappa = 2L\rangle$

	L_1	$\lambda - L =$ even
$l = 0$	L	$\sqrt{\frac{8(L-1)L(L+1)(L+2)}{15(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
$l = 2$	$L - 2$	$(\lambda - 5L + 8)\sqrt{\frac{(L+1)(L+2)(\lambda-L+2)(\lambda+L+1)}{7(2L-1)(2L+1)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
	L	$(3\lambda^2 + 21\lambda + 30 - 7L - 7L^2)\sqrt{\frac{2(L-1)(L+2)}{21(2L-1)(2L+3)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L + 2$	$(\lambda + 5L + 13)\sqrt{\frac{(L-1)L(\lambda-L)(\lambda+L+3)}{7(2L+1)(2L+3)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
$l = 4$	$L - 4$	$\sqrt{\frac{6(L-3)(L-2)(L+1)(L+2)(\lambda-L+2)(\lambda-L+4)(\lambda+L-1)(\lambda+L+1)}{(2L-5)(2L-3)(2L-1)(2L+1)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L - 2$	$(-33 - L + 4L^2 - 12\lambda + 2\lambda L)\sqrt{\frac{6(L-2)(L+2)(\lambda-L+2)(\lambda+L+1)}{7(2L-5)(2L+3)(2L-1)(2L+1)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
	L	$-(10L^2\lambda^2 + 10L\lambda^2 - 90\lambda^2 + 70L^2\lambda + 70L\lambda - 420\lambda - 14L^4 - 28L^3 + 149L^2 + 163L - 480)$
		$\times\sqrt{\frac{3}{35(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L + 2$	$(2L\lambda + 14\lambda + 28 - 9L - 4L^2)\sqrt{\frac{6(L-1)(L+3)(\lambda-L)(\lambda+L+3)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$
	$L + 4$	$\sqrt{\frac{6(L-1)L(L+3)(L+4)(\lambda-L-2)(\lambda-L)(\lambda+L+3)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-1)\lambda(2\lambda^2+8\lambda-L^2-L+8)}}$

	L_1	$\lambda - L =$ odd
$l = 2$	$L - 1$	$-(\lambda - 3L + 5)\sqrt{\frac{(L+2)(\lambda+L+2)}{7(\lambda-1)\lambda(\lambda+2)(2L+1)}}$
	$L + 1$	$-(\lambda + 3L + 8)\sqrt{\frac{(L-1)(\lambda-L+1)}{7(2L+1)(\lambda-1)\lambda(\lambda+2)}}$
$l = 4$	$L - 3$	$-\sqrt{\frac{3(L-3)(L-2)(L+2)(\lambda-L+3)(\lambda+L)(\lambda+L+2)}{(2L-3)(2L-1)(2L+1)(\lambda-1)\lambda(\lambda+2)}}$
	$L - 1$	$(L\lambda + 9\lambda + 24 - L - 3L^2)\sqrt{\frac{6(L-2)(\lambda+L+2)}{7(2L-3)(2L+1)(2L+3)(\lambda-1)\lambda(\lambda+2)}}$
	$L + 1$	$(L\lambda - 8\lambda - 22 + 5L + 3L^2)\sqrt{\frac{6(L+3)(\lambda-L+1)}{7(2L-1)(2L+1)(2L+5)(\lambda-1)\lambda(\lambda+2)}}$
	$L + 3$	$-\sqrt{\frac{3(L-1)(L+3)(L+4)(\lambda-L-1)(\lambda-L+1)(\lambda+L+4)}{(2L+1)(2L+3)(2L+5)(\lambda-1)\lambda(\lambda+2)}}$

Table 4. $\langle(\lambda,0)L_1;(4,0)l\|(\lambda-2,3)\kappa L\rangle$.

		$\langle(\lambda,0)L_1;(4,0)l\|(\lambda-2,3)\kappa=1L\rangle$
	L_1	$\lambda-L=\text{even}$
$l=0$	L	$\sqrt{\frac{4L(L+1)(2-3L-3L^2+4\lambda^2)}{15(\lambda-2)\lambda(\lambda+1)(\lambda+2)}}$
$l=2$	$L-2$	$(-8+6L^2+6\lambda-3L\lambda-4\lambda^2)\sqrt{\frac{2(L-1)(L+1)(\lambda-L+2)(\lambda+L+1)}{7(2L-1)(2L+1)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
	L	$\frac{(60-50L-38L^2+24L^3+12L^4+6\lambda-9L\lambda-9L^2\lambda+12\lambda^2-10L\lambda^2-10L^2\lambda^2+12\lambda^3)}{\sqrt{21(2L-1)(2L+3)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
	$L+2$	$(2-12L-6L^2-9\lambda-3L\lambda+4\lambda^2)\sqrt{\frac{2L(L+2)(\lambda-L)(\lambda+L+3)}{7(2L+1)(2L+3)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
$l=4$	$L-4$	$(\lambda-L-1)\sqrt{\frac{12(L-3)(L-2)(L-1)(L+1)(\lambda+L-1)(\lambda+L+1)(\lambda-L+2)(\lambda-L+4)}{(2L-5)(2L-3)(2L-1)(2L+1)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
	$L-2$	$(66+38L-4L^2-4L^3+3\lambda+5L\lambda+2L^2\lambda-9\lambda^2-2L\lambda^2)$
		$\times\sqrt{\frac{3(L-2)(L-1)(\lambda-L+2)(\lambda+L+1)}{7(2L-5)(2L-1)(2L+1)(2L+3)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
	L	$\frac{(-240+74L+68L^2-12L^3-6L^4-150\lambda+15\lambda L+15\lambda L^2+15\lambda^2-2L\lambda^2-2L^2\lambda^2+15\lambda^3)\sqrt{6(L-1)(L+2)}}{\sqrt{35(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
	$L+2$	$(28-34L+8L^2+4L^3-L\lambda+2L^2\lambda-7\lambda^2+2\lambda^2L)$
		$\times\sqrt{\frac{3(L+2)(L+3)(\lambda-L)(\lambda+L+3)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-2)\lambda(\lambda+1)(2-3L-3L^2+4\lambda^2)}}$
	$L+4$	$-(\lambda+L)\sqrt{\frac{12L(L+2)(L+3)(L+4)(\lambda-L-2)(\lambda-L)(\lambda+L+3)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-3L-3L^2+4\lambda^2)}}$
	L_1	$\lambda-L=\text{odd}$
$l=2$	$L-1$	$(4\lambda^2+2L\lambda-4\lambda+10-L-3L^2)\sqrt{\frac{(L-1)(\lambda+L+2)}{7(2L+1)(\lambda-2)\lambda(\lambda+1)(2-L-L^2+4\lambda^2)}}$
	$L+1$	$-(4\lambda^2-2\lambda L-6\lambda+8-5L-3L^2)\sqrt{\frac{(L+2)(\lambda-L+1)}{7(2L+1)(\lambda-2)\lambda(\lambda+1)(2-L-L^2+4\lambda^2)}}$
$l=4$	$L-3$	$-(\lambda-L-2)\sqrt{\frac{3(L-3)(L-2)(L-1)(\lambda-L+3)(\lambda+L)(\lambda+L+2)}{(2L-3)(2L-1)(2L+1)(\lambda-2)\lambda(\lambda+1)(\lambda+2)(2-L-L^2+4\lambda^2)}}$
	$L-1$	$(-24+L+3L^2-3\lambda-2L\lambda+3\lambda^2)\sqrt{\frac{3(L-2)(L-1)(L+2)(\lambda+L+2)}{7(2L-3)(2L+1)(2L+3)(\lambda-2)\lambda(\lambda+1)(2-L-L^2+4\lambda^2)}}$
	$L+1$	$(22-5L-3L^2+\lambda-2L\lambda-3\lambda^2)\sqrt{\frac{3(L-1)(L+2)(L+3)(\lambda-L+1)}{7(2L-1)(2L+1)(2L+5)(\lambda-2)\lambda(\lambda+1)(2-L-L^2+4\lambda^2)}}$
	$L+3$	$(\lambda+L-1)\sqrt{\frac{3(L+2)(L+3)(L+4)(\lambda-L-1)(\lambda-L+1)(\lambda+L+4)}{(2L+1)(2L+3)(2L+5)(\lambda-2)\lambda(\lambda+1)(2-L-L^2+4\lambda^2)}}$
		$\langle(\lambda,0)L_1;(4,0)l\|(\lambda-2,3)\kappa=3L\rangle$
	L_1	$\lambda-L=\text{even}$
$l=0$	L	0
$l=2$	$L-2$	$\sqrt{\frac{6(L-2)(L+1)(L+2)(L+3)(\lambda-L)(\lambda-L+2)}{7(2L-1)(2L+1)(\lambda-2)\lambda(2-3L-3L^2+4\lambda^2)}}$
	L	$\sqrt{\frac{36(L-2)(L-1)(L+2)(L+3)(\lambda-L)(\lambda+L+1)}{7(2L-1)(2L+3)(\lambda-2)\lambda(2-3L-3L^2+4\lambda^2)}}$
	$L+2$	$\sqrt{\frac{6(L-2)(L-1)L(L+3)(\lambda+L+1)(\lambda+L+3)}{7(2L+1)(2L+3)(\lambda-2)\lambda(2-3L-3L^2+4\lambda^2)}}$
$l=4$	$L-4$	$-\sqrt{\frac{4(L-3)(L+1)(L+2)(L+3)(\lambda-L)(\lambda-L+2)(\lambda-L+4)(\lambda+L-1)}{(2L-5)(2L-3)(2L-1)(2L+1)\lambda(\lambda-2)(2-3L-3L^2+4\lambda^2)}}$
	$L-2$	$(46+30L+49\lambda-14\lambda L-16L^2)\sqrt{\frac{(L+2)(L+3)(\lambda-L)(\lambda-L+2)}{7(2L-5)(2L-1)(2L+1)(2L+3)(\lambda-2)\lambda(4\lambda^2+2-3L-3L^2)}}$
	L	$(7\lambda-2L^2-2L+11)\sqrt{\frac{90(L-2)(L+3)(\lambda-L)(\lambda+L+1)}{7(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-2)\lambda(2-3L-3L^2+4\lambda^2)}}$
	$L+2$	$(-62L-16L^2+63\lambda+14L\lambda)\sqrt{\frac{(L-2)(L-1)(\lambda+L+1)(\lambda+L+3)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-2)\lambda(2-3L-3L^2+4\lambda^2)}}$
	$L+4$	$\sqrt{\frac{4(L-2)(L-1)L(L+4)(\lambda-L-2)(\lambda+L+1)(\lambda+L+3)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-2)\lambda(2-3L-3L^2+4\lambda^2)}}$

Table 4. (Continued)

		$\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda-2, 3)\kappa = 1L\rangle$
	L_1	$\lambda - L = \text{odd}$
$l=2$	$L-1$	$-\sqrt{\frac{12(L-2)(L+2)(L+3)(\lambda-L+1)}{7(2L+1)(\lambda-2)(2-L-L^2+4\lambda^2)}}$
	$L+1$	$-\sqrt{\frac{12(L-2)(L-1)(L+3)(\lambda+L+2)}{7(2L+1)(\lambda-2)(2-L-L^2+4\lambda^2)}}$
$l=4$	$L-3$	$\sqrt{\frac{9(L-3)(L+2)(L+3)(\lambda-L+1)(\lambda-L+3)(\lambda+L)}{(2L-3)(2L-1)(2L+1)(\lambda-2)(2-L-L^2+4\lambda^2)}}$
	$L-1$	$-(66+7L-13L^2+42\lambda-7L\lambda)\sqrt{\frac{(L+3)(\lambda-L+1)}{7(2L-3)(2L+1)(2L+3)(\lambda-2)(2-L-L^2+4\lambda^2)}}$
	$L+1$	$-(46-33L-13L^2+49\lambda+7L\lambda)\sqrt{\frac{(L-2)(\lambda+L+2)}{7(2L-1)(2L+1)(2L+5)(\lambda-2)(2-L-L^2+4\lambda^2)}}$
	$L+3$	$-\sqrt{\frac{9(L-2)(L-1)(L+4)(\lambda-L-1)(\lambda+L+2)(\lambda+L+4)}{(2L+1)(2L+3)(2L+5)(\lambda-2)(2-L-L^2+4\lambda^2)}}$

The building-up method of Vergados [18] has been used. Specifically, equation (14) of [18] is used in calculating the Wigner coefficients

$$\begin{aligned}
&\langle(\lambda_1, \mu_1)\kappa_1 L_1; (\lambda_{23}, \mu_{23})\kappa_{23} L_{23}\|(\lambda, \mu)\kappa L\rangle \\
&\qquad \times U((\lambda_1, \mu_1)(\lambda_2, \mu_2)(\lambda, \mu)(\lambda_3, \mu_3); (\lambda_{12}, \mu_{12})(\lambda_{23}, \mu_{23})) \\
&\quad = \sum_{\kappa_{12} L_{12} l_2 l_3} \langle(\lambda_1, \mu_1)\kappa_1 L_1; (\lambda_2, \mu_2)\kappa_2 L_2\|(\lambda_{12}, \mu_{12})\kappa_{12} L_{12}\rangle \\
&\qquad \times \langle(\lambda_2, \mu_2)\kappa_2 L_2; (\lambda_3, \mu_3)\kappa_3 L_3\|(\lambda_{23}, \mu_{23}\kappa_{23} L_{23}\rangle \\
&\qquad \times \langle(\lambda_{12}, \mu_{12})\kappa_{12} L_{12}; (\lambda_3, \mu_3)\kappa_3 L_3\|(\lambda\mu)\kappa L\rangle U(L_1 L_2 L L_3; L_{12} L_{23}).
\end{aligned} \tag{1}$$

We have chosen $(\lambda_1, \mu_1) = (\lambda, 0)$, $(\lambda_2, \mu_2) = (3, 0)$ and $(\lambda_3, \mu_3) = (1, 0)$. $(\lambda_{23}, \mu_{23}) = (4, 0)$. (λ_{12}, μ_{12}) is different for different (λ, μ)'s. We have the following: (1) for $(\lambda, \mu) = (\lambda_1 + 4, 0)$, $(\lambda_{12}, \mu_{12}) = (\lambda_1 + 3, 0)$; (2) for $(\lambda, \mu) = (\lambda_1 + 2, 1)$, $(\lambda_{12}, \mu_{12}) = (\lambda_1 + 3, 0)$ or $(\lambda_1 + 1, 1)$; (3) for $(\lambda\mu) = (\lambda_1 2)$, $(\lambda_{12}\mu_{12}) = (\lambda_1 + 11)$ or $(\lambda_1 - 1, 2)$; (4) for $(\lambda, \mu) = (\lambda_1 - 2, 3)$, $(\lambda_{12}, \mu_{12}) = (\lambda_1 - 1, 2)$ or $(\lambda_1 - 3, 3)$; (5) for $(\lambda, \mu) = (\lambda_1 - 4, 4)$, $(\lambda_{12}, \mu_{12}) = (\lambda_1 - 3, 3)$.

During the calculation, we first obtain the unnormalized Wigner coefficients defined as $\langle(\lambda_1, \mu_1)\kappa_1 L_1; (\lambda_2\mu_2)\kappa_2 L_2\|(\lambda_3, \mu_3)\kappa_3 L_3\rangle \times U((\lambda_1, \mu_1)(\lambda_2, \mu_2)(\lambda, \mu)(\lambda_3, \mu_3); (\lambda_{12}, \mu_{12})(\lambda_{23}, \mu_{23}))$ for all possible $\kappa_1 L_1$ and $\kappa_2 L_2$. The sum of all the unnormalized Wigner coefficients squared for a given $(\lambda\mu)$ should give the square of the $SU(3)$ U-function, because

$$\sum_{\kappa_1 L_1 \kappa_2 L_2} \langle(\lambda_1\mu_1)\kappa_1 L_1; (\lambda_2\mu_2)\kappa_2 L_2)\|(\lambda\mu)\kappa L\rangle^2 = 1. \tag{2}$$

This provides a rigorous check on the Wigner coefficients. With the choice of phase convention in Vergados basis, the $SU(3)$ U-function is real, and its sign can be determined. The $SU(3)$ U-function obtained in the present calculation is given in table 1.

Most of the coefficients needed can be found in [18], and the $(\lambda, 0) \times (3, 0)$ is given in [26]. The Wigner coefficients $\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda + 2, 1)\rangle$, $\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda, 2)\rangle$, $\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda - 2, 3)\rangle$, $\langle(\lambda, 0)L_1; (4, 0)l\|(\lambda - 4, 4)\rangle$, are given in tables 2–5 respectively. For cross-checking, we have also calculated the coefficients by different route of building-up, where we have used $(\lambda_1, \mu_1) = (\lambda, 0)$, $(\lambda_2, \mu_2) = (2, 0)$ and $(\lambda_3, \mu_3) = (2, 0)$ and $(\lambda_{23}, \mu_{23}) = (4, 0)$. The same results have been obtained. The corresponding U functions are also included in table 1.

Table 5. $\langle(\lambda,0)L_1;(4,0)l\|(\lambda-4,4)\kappa L\rangle$.

		$\langle(\lambda,0)L_1;(4,0)l\|(\lambda-4,4)\kappa=0L\rangle$, where $P(\lambda,L)=8(\lambda+2)^2(\lambda+4)^2-(8\lambda^2+48\lambda+82)L(L+1)+3L^2(L+1)^2$
	L_1	$\lambda-L=$ even
$l=0$	L	$\sqrt{\frac{(-18L-15L^2+6L^3+3L^4+16L\lambda+16L^2\lambda+32\lambda^2-8L\lambda^2-8L^2\lambda^2-32\lambda^3+8\lambda^4)P(\lambda-4,L)}{15(\lambda-2)(\lambda-1)\lambda(\lambda+1)}}$
$l=2$	$L-2$	$-(\lambda-L-3)(6+3L-3L^2-8\lambda+4\lambda^2)\sqrt{\frac{2(L-1)L(\lambda-L+2)(\lambda+L+1)}{7(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
	L	$(-54+27L+24L^2-6L^3-3L^4+36\lambda-10L\lambda-10L^2\lambda-2\lambda^2+5L\lambda^2+5L^2\lambda^2-16\lambda^3+4\lambda^4)\sqrt{\frac{4L(L+1)}{21(2L-1)(2L+3)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
	$L+2$	$(\lambda+L-2)(9L+3L^2+8\lambda-4\lambda^2)\sqrt{\frac{2(L+1)(L+2)(\lambda-L)(\lambda+L+3)}{7(2L+1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
$l=4$	$L-4$	$(\lambda-L-3)(\lambda-L-1)\sqrt{\frac{3(L-3)(L-2)(L-1)L(\lambda-L+2)(\lambda-L+4)(\lambda+L-1)(\lambda+L+1)}{(2L-5)(2L-3)(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
	$L-2$	$-(\lambda-L-3)(-9-L+L^2-2\lambda+\lambda^2)\sqrt{\frac{12(L-2)(L-1)L(L+1)(\lambda-L+2)(\lambda+L+1)}{7(2L-5)(2L-1)(2L+1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
	L	$(180-48L-45L^2+6L^3+3L^4+90\lambda-4L\lambda-4L^2\lambda-33\lambda^2+2L\lambda^2+2L^2\lambda^2-12\lambda^3+3\lambda^4)\sqrt{\frac{6(L-1)L(L+1)(L+2)}{35(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
	$L+2$	$-(\lambda+L-2)(-7+3L+L^2-2\lambda+\lambda^2)\sqrt{\frac{12L(L+1)(L+2)(L+3)(\lambda-L)(\lambda+L+3)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
	$L+4$	$(\lambda+L-2)(\lambda+L)\sqrt{\frac{3(L+1)(L+2)(L+3)(L+4)(\lambda-L-2)(\lambda-L)(\lambda+L+3)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-2)(\lambda-1)\lambda(\lambda+1)P(\lambda-4,L)}}$
		$\langle(\lambda,0)L_1;(4,0)l\|(\lambda-4,4)\kappa=2L\rangle$, where $Q(\lambda,L)=2(\lambda+5)^2(\lambda^2+4\lambda+6)-2(\lambda^2+7\lambda+16)L-(2\lambda^2+14\lambda+31)L^2+2L^3+L^4$
	L_1	$\lambda-L=$ even
$l=0$	L	$(2\lambda^2-4\lambda-L^2-L+6)\sqrt{\frac{(L-1)L(L+1)(L+2)}{5(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
$l=2$	$L-2$	$(6-7L+L^3+2\lambda+4L\lambda-L^2\lambda-3\lambda^2-L\lambda^2+\lambda^3)\times\sqrt{\frac{6(L+1)(L+2)(\lambda-L+2)(\lambda+L+1)}{7(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
	L	$(18-9L-8L^2+2L^3+L^4+24\lambda+L\lambda+L^2\lambda-3\lambda^2-2L\lambda^2-2L^2\lambda^2-6\lambda^3+3\lambda^4)\sqrt{\frac{4(L-1)(L+2)}{7(2L-1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
	$L+2$	$(12+4L-3L^2-L^3-3\lambda-6L\lambda-L^2\lambda-2\lambda^2+L\lambda^2+\lambda^3)\times\sqrt{\frac{6(L-1)L(\lambda-L)(\lambda+L+3)}{7(2L+1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
$l=4$	$L-4$	$-(\lambda-L-3)(\lambda-L+1)\sqrt{\frac{(L-3)(L-2)(L+1)(L+2)(\lambda-L+2)(\lambda-L+4)(\lambda+L-1)(\lambda+L+1)}{(2L-5)(2L-3)(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
	$L-2$	$(-27+15L+14L^2-L^3-L^4-51\lambda+L\lambda+4L^2\lambda+L^3\lambda-18\lambda^2-3L\lambda^2+L^2\lambda^2+6\lambda^3-L\lambda^3)\sqrt{\frac{4(L-2)(L+2)(\lambda-L+2)(\lambda+L+1)}{7(2L-5)(2L-1)(2L+1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
	L	$(360-276L-222L^2+105L^3+45L^4-9L^5-3L^6+900\lambda-311L\lambda-293L^2\lambda+36L^3\lambda+18L^4\lambda+675\lambda^2-122L\lambda^2-116L^2\lambda^2+12L^3\lambda^2+6L^4\lambda^2+90\lambda^3-10L\lambda^3-10L^2\lambda^3-45\lambda^4+5L\lambda^4+5L^2\lambda^4)\sqrt{\frac{2}{35(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
	$L+2$	$(28-12L-11L^2+3L^3+L^4+49\lambda-4L\lambda-L^2\lambda+L^3\lambda+14\lambda^2-5L\lambda^2-L^2\lambda^2-7\lambda^3-L\lambda^3)\sqrt{\frac{4(L-1)(L+3)(\lambda-L)(\lambda+L+3)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$
	$L+4$	$-(\lambda+L-2)(\lambda+L+2)\sqrt{\frac{(L-1)L(L+3)(L+4)(\lambda-L-2)(\lambda-L)(\lambda+L+3)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-2)(\lambda-1)\lambda(\lambda+1)Q(\lambda-4,L)}}$

Table 5. (Continued)

		$\lambda - L = \text{odd}$
$l=2$	$L-1$	$-\sqrt{\frac{3(L+2)(\lambda-L+1)(12-L-L^2-8\lambda+2\lambda^2)}{7(2L+1)(\lambda-2)(\lambda-1)\lambda}}$
	$L+1$	$-\sqrt{\frac{3(L-1)(\lambda+L+2)(12-L-L^2-8\lambda+2\lambda^2)}{7(2L+1)(\lambda-2)(\lambda-1)\lambda}}$
$l=4$	$L-3$	$(\lambda-L+4)\sqrt{\frac{(L-3)(L-2)(L+2)(\lambda-L+1)(\lambda-L+3)(\lambda+L)}{(2L-3)(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(12-L-L^2-8\lambda+2\lambda^2)}}$
	$L-1$	$-(-72-30L+9L^2+3L^3-36\lambda-4L\lambda+9\lambda^2+L\lambda^2)$ $\times\sqrt{\frac{(L-2)(\lambda-L+1)}{7(2L-3)(2L+1)(2L+3)(\lambda-2)(\lambda-1)(12-L-L^2-8\lambda+2\lambda^2)}}$
	$L+1$	$-(36-39L+3L^3+32\lambda-4L\lambda-8\lambda^2+L\lambda^2)$ $\times\sqrt{\frac{(L+3)(\lambda+L+2)}{7(2L-1)(2L+1)(2L+5)(\lambda-2)(\lambda-1)\lambda(12-L-L^2-8\lambda+2\lambda^2)}}$
	$L+3$	$(\lambda+L-3)\sqrt{\frac{(L-1)(L+3)(L+4)(\lambda+L-1)(\lambda+L+2)(\lambda+L+4)}{(2L+1)(2L+3)(2L+5)(\lambda-2)(\lambda-1)\lambda(12-L-L^2-8\lambda+2\lambda^2)}}$
		$\langle(\lambda,0)L_1;(4,0)i\Vert(\lambda-4,4)\kappa=4L\rangle$, where $R(\lambda,L)=(\lambda-L+1)(\lambda-L+9)(\lambda-L+7)(\lambda-L+5)$ $+28(\lambda-L+9)(\lambda-L+7)(\lambda-L+5)(\lambda+L+5)+70(\lambda-L+7)(\lambda-L+5)(\lambda+L+3)(\lambda+L+5)$ $+28(\lambda-L+5)(\lambda+L+1)(\lambda+L+3)(\lambda+L+5)+(\lambda+L-1)(\lambda+L+1)(\lambda+L+3)(\lambda+L+5)$
	L_1	$\lambda - L = \text{even}$
$l=0$	L	$\sqrt{\frac{(L-3)(L-2)(L-1)L(L+1)(L+2)(L+3)(L+4)}{5(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
$l=2$	$L-2$	$(\lambda-L+1)\sqrt{\frac{6(L-3)(L-2)(L+1)(L+2)(L+3)(L+4)(\lambda-L+2)(\lambda+L+1)}{7(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	L	$(3-L-L^2+6\lambda+3\lambda^2)\sqrt{\frac{4(L-3)(L-2)(L-1)(L+2)(L+3)(L+4)}{7(2L-1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	$L+2$	$(\lambda+L+2)\sqrt{\frac{6(L-3)(L-2)(L-1)L(L+3)(L+4)(\lambda-L)(\lambda+L+3)}{7(2L+1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
$l=4$	$L-4$	$(\lambda-L+3)(\lambda-L+1)\sqrt{\frac{(L+1)(L+2)(L+3)(L+4)(\lambda-L+2)(\lambda-L+4)(\lambda+L-1)(\lambda+L+1)}{(2L-5)(2L-3)(2L-1)(2L+1)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	$L-2$	$(9-8L-2L^2+L^3+23\lambda-13L\lambda-L^2\lambda+21\lambda^2-7L\lambda^2+7\lambda^3)$ $\times\sqrt{\frac{4(L-3)(L+2)(L+3)(L+4)(\lambda-L+2)(\lambda+L+1)}{7(2L-5)(2L-1)(2L+1)(2L+3)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	L	$(60-36L-33L^2+6L^3+3L^4+190\lambda-60L\lambda-60L^2\lambda+235\lambda^3-30L\lambda^2-30L^2\lambda^2+140\lambda^3$ $+35\lambda^4)\sqrt{\frac{2(L-3)(L-2)(L+3)(L+4)}{35(2L-3)(2L-1)(2L+3)(2L+5)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	$L+2$	$(14+L-5L^2-L^3+35\lambda+11L\lambda-L^2\lambda+28\lambda^2+7L\lambda^2+7\lambda^3)$ $\times\sqrt{\frac{4(L-3)(L-2)(L-1)(L+4)(\lambda-L)(\lambda+L+3)}{7(2L-1)(2L+1)(2L+3)(2L+7)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	$L+4$	$(\lambda+L+2)(\lambda+L+4)\sqrt{\frac{(L-3)(L-2)(L-1)L(\lambda-L-2)(\lambda-L)(\lambda+L+3)(\lambda+L+5)}{(2L+1)(2L+3)(2L+5)(2L+7)(\lambda-2)(\lambda-1)\lambda(\lambda+1)R(\lambda-4,L)}}$
	L_1	$\lambda - L = \text{odd}$
$l=2$	$L-1$	0
	$L+1$	0
$l=4$	$L-3$	$-\sqrt{\frac{(L+2)(L+3)(L+4)(\lambda-L-1)(\lambda-L+1)(\lambda-L+3)}{(2L-3)(2L-1)(2L+1)(\lambda-2)(12-L-L^2-8\lambda+2\lambda^2)}}$
	$L-1$	$-\sqrt{\frac{7(L-3)(L+3)(L+4)(\lambda-L-1)(\lambda-L+1)(\lambda+L)}{(2L-3)(2L+1)(2L+3)(\lambda-2)(12-L-L^2-8\lambda+2\lambda^2)}}$
	$L+1$	$-\sqrt{\frac{7(L-3)(L-2)(L+4)(\lambda-L-1)(\lambda+L)(\lambda+L+2)}{(2L-1)(2L+1)(2L+5)(\lambda-2)(12-L-L^2-8\lambda+2\lambda^2)}}$
	$L+3$	$-\sqrt{\frac{(L-3)(L-2)(L-1)(\lambda+L)(\lambda+L+2)(\lambda+L+4)}{(2L+1)(2L+3)(2L+5)(\lambda-2)(12-L-L^2-8\lambda+2\lambda^2)}}$

The substitution property found for $R_5 \supset R_3$ [27] and for $SU_3 \supset R_3$ Wigner coefficients in [26] are also present for $(\lambda, 0) \times (4, 0)$ case. $\langle(\lambda_1 0)L-q;(40)l\Vert(\lambda\mu)\kappa L\rangle$

and $\langle(\lambda_1 0)L+q;(40)l\|(\lambda\mu)\kappa L\rangle$ can be obtained from one another by the substitution $L\to-(L+1)$ apart from an overall sign. The origin of this property lies in the mirror symmetry of the $6j$ symbol [28].

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